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SOME RECENT DEVELOPMENTS IN THE STUDY OF EDGE VORTICES

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INTRODUCTION

The aerodynamic properties of a slender winglike body are greatly influenced by a vortex system which is associated with boundary-layer separation at the leading or side edges of the body. The broad features of the flow are reasonably well understood. And the evolution of an aerodynamic design technique, which makes at least some provision for the vortex growth, has been in process for several years.[1] But the considerable progress that has been made in this aspect of slender wing research[2-3] has been achieved with only a rudimentary understanding of the mechanics of vortex structure and growth. What has been done so far has been based on calculation at the so-called design point—at which there are, in fact, no vortices—under certain restrictions aimed at establishing suitable starting conditions for the vortex system. But physical reasoning has played a part both as the source of these restrictions and as a guide in the largely empirical development of acceptable "off-design" behavior. And I believe that its influence is likely to increase.

In my view, therefore, there is a strong practical case for research concerned primarily with the generation of physical ideas on the evolution of edge vortices. And I believe that enough has been done already to justify an attempt to outline the sort of physical picture that is beginning to emerge, and to indicate possibly fruitful areas for future study. I believe, also, that most of the ideas derived from this work, which has been mainly concerned with the quasi-conical vortex generated by a slender sharp-edged conical body at incidence, are likely to prove at least partially relevant to a wide class of flows which involve coherent edge vortex systems—like, for example, the initial period in the transient development of a bluff-body wake.

SOME EXPERIMENTAL OBSERVATIONS

The flow within the core of a leading edge vortex has been studied extensively. But only during the last few years have experimental surveys begun to reveal the finer details of the core structure. The most striking feature of the earlier work was the high velocity—approaching twice that of the undisturbed stream—observed by Kirby[4] along the axis of a vortex shed from the leading edge of a delta wing of 9° semi-apex angle at 25° incidence. This could be explained qualitatively in terms of the familiar spiral sheet model of the vortex. For the inclination of the spiralling vortex lines to the axis is such as to make them all induce a downstream component of velocity along the axis. But the model is plainly invalid at finite Reynolds numbers, since it leads to an infinite axial velocity. And the measurements themselves were open to some doubt on the grounds that the magnitude of the systematic error incurred through the use of a five-tube yawmeter head was uncertain.

Kirby's results were therefore regarded at the time as somewhat inconclusive. But they stimulated interest in the vortex core as a subject of study on its own account. And they were followed by a succession of attempts to measure the axial velocity, embracing a wide range of experimental techniques. For example, Cox[5] measured the velocity of puffs of smoke released in a vortex at low speeds; Lambourne and Bryer[6] employed conventional pitot and static tubes, carefully aligned along the axis of the vortex, in air, and also measured the velocity of filaments of dye introduced into a vortex in water. Earnshaw[7] again used a five-tube yawmeter head. But he has followed this up very recently[8] with some check measurements using a version of the spark discharge technique originally introduced by Townend[9] for use in boundary layers. Figure 1 shows an example of the results he has obtained. It is a photograph of a train of electric sparks which were caused to jump between two electrodes arranged so as to make the first spark pass as nearly as possible through the axis of the vortex. This first spark leaves a column of ionized air behind it, which is convected downstream, and

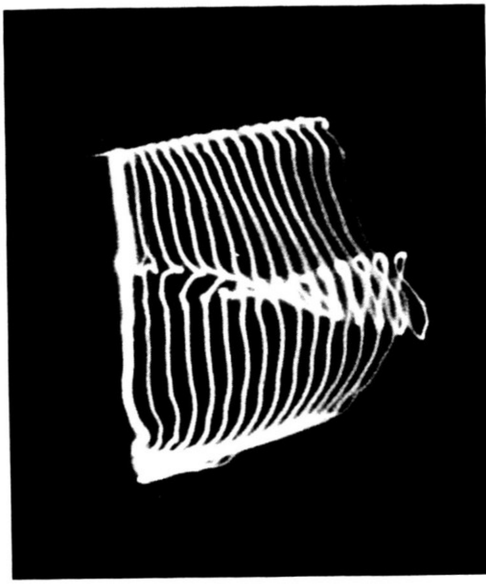

Fig. 1. Train of electric sparks discharged across the core of a vortex. (Crown copyright reserved.)

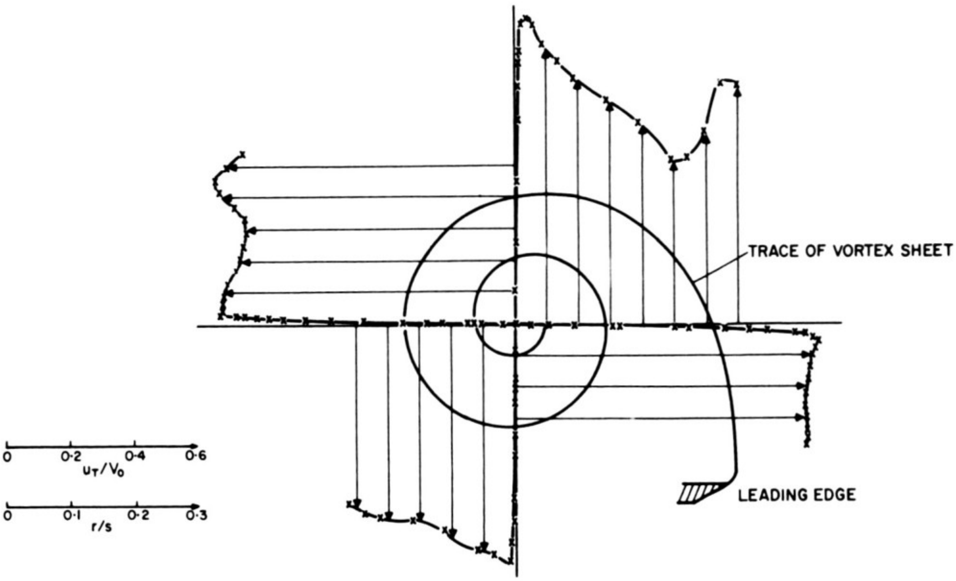

Fig. 2.

is then reilluminated by the subsequent sparks. These tend to pass through the same set of gas particles, because they offer the path of least resistance between the electrodes. We therefore obtain a succession of effectively instantaneous pictures of the original column of air particles, distorted by the vortex motion.

This later work has both confirmed Kirby's earlier observations and given grounds for greater confidence in the five-tube yawmeter head. An axial velocity of very nearly five times the free stream velocity has now been measured, and there is no sign of a limit yet. For both Earnshaw and Lambourne and Bryer have found that the axial velocity increases with increasing Reynolds number throughout the range of their experiments.

Earnshaw's work, in particular, was specifically designed to investigate the small-scale structure of a vortex. His measurements were very closely spaced in two mutually perpendicular traverses in each of three chordwise stations. And he found a succession of kinks in the circumferential velocity distribution, in the outer region of the vortex, which he attributed to the residual effect of the spiral structure (see Fig. 2). He also found the region of effective solid body rotation near the axis of the core to be exceedingly small—so small, in fact, that most previous investigations had missed it altogether.

THE VORTEX CORE

The core studied by Earnshaw appears to have been slightly elliptic in cross section with, somewhat surprisingly perhaps, its major axis normal to the wing surface. But by assuming the departure from axial symmetry within the core to be negligibly small, and noting that the residual effects of the spiral structure are negligible over a substantial part of the vortex, Hall[10] has been able to develop a theory which allows him to predict most of the observed properties of the core region, at least qualitatively.

I cannot go into the details of Hall's theory. But broadly what he does is to represent the core by a continuous distribution of vorticity with negligibly small diffusion in its outer part. He then makes use of approximations of essentially boundary-layer type to find a compatible inner solution. And in this way, he demonstrates that the given external conditions imply the existence of an inner subcore, the structure of which is dominated by viscous diffusion. His results are in good qualitative agreement with experiment, as Figs. 3-5 show. And since the Reynolds numbers of the experiments available for comparison are rather low for the approximations in the theory to be strictly valid, the quantitative differences may not be very significant.

However, although Hall's theory sheds considerable light on the mechanism governing the structure of the inner subcore, the nature of the outer core region is assumed. The assumption is justified by appeal to experiment. But it has not

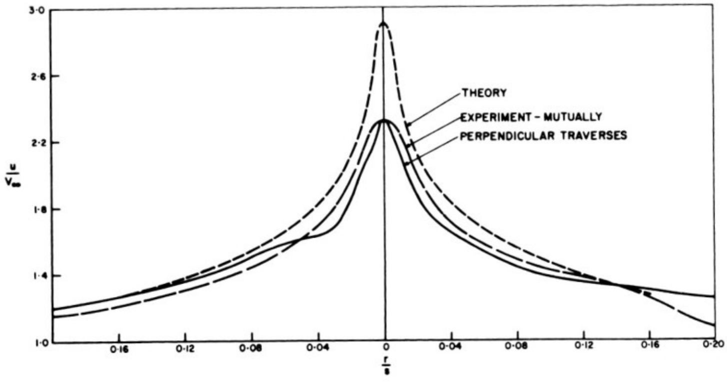

Fig. 3.

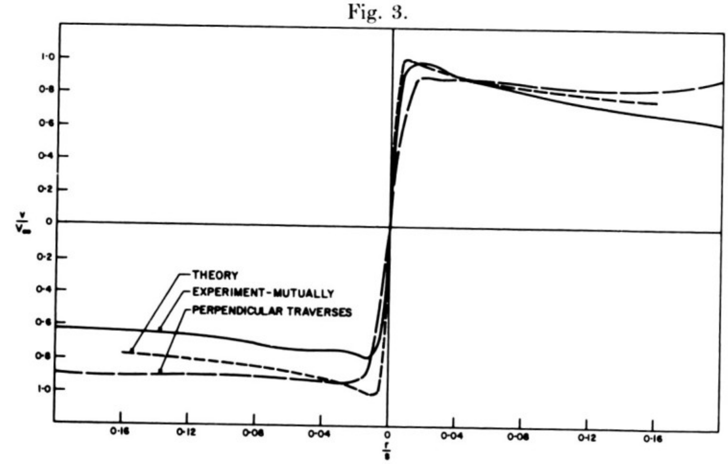

Fig. 4.

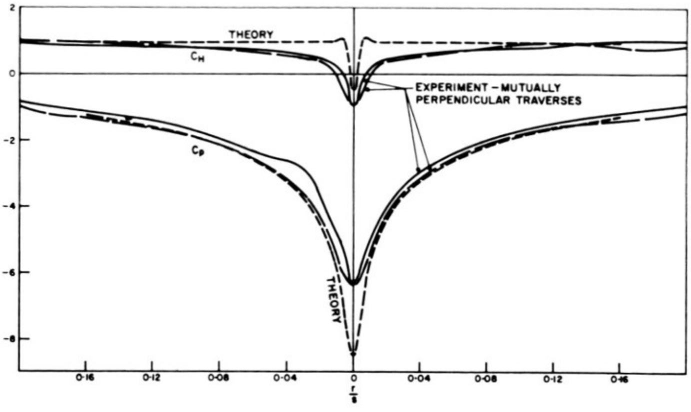

Fig. 5.

yet been supported by physical argument. And to be convincing, such an argument must be part of a unified physical interpretation of the observed vortex structure, embracing the whole of the vortex from the outer spiral region to the inner diffusive subcore.

A PHYSICAL PICTURE OF THE VORTEX

It seems to me to be possible to explain the main features of the observed vortex structure in quite simple terms. In a viscous fluid vorticity is transported by convection and diffusion. But there is a marked difference in the scale of these two transport processes at high Reynolds numbers. In time t an element of vorticity is carried with the fluid a distance x (equal to Ut, where U is the average velocity at the element during the interval) and, at the same time, it diffuses outwards a distance of order $(\nu t)^{1/2}$. Thus the ratio of the scales of diffusion and convection is $0(\nu/Ux)^{1/2}$. But the rate of expansion of the vortex with distance x is $0(l/x)$, where l is the lateral scale of the vortex as a whole—i.e., $l(x)$ is any typical dimension of the large-scale structure of the vortex in the plane x = const., the origin of x being chosen so as to make $l(0) = 0$. And in the vortices studied experimentally $l/x \gg (\nu/Ux)^{1/2}$. So it appears that in these cases the large-scale vortex structure must have been determined primarily by the convective transport mechanism, and is likely to have been largely independent of Reynolds number.

However, convection, though essentially a large-scale process, also partially determines the small-scale structure of the vortex. For consider the situation at infinite Reynolds number, where there is no diffusion, and the vortex is consequently formed solely by convection. In this case a vortex sheet, shed from the leading edge of the wing, curls up into an expanding spiral stream surface above the wing (see Fig. 6). The form of the spiral trace that this surface makes in any plane x = const. normal to its axis may be defined by the spacing d between successive turns of the spiral. The length d is therefore a local measure of an essentially small-scale structure associated solely with convection.

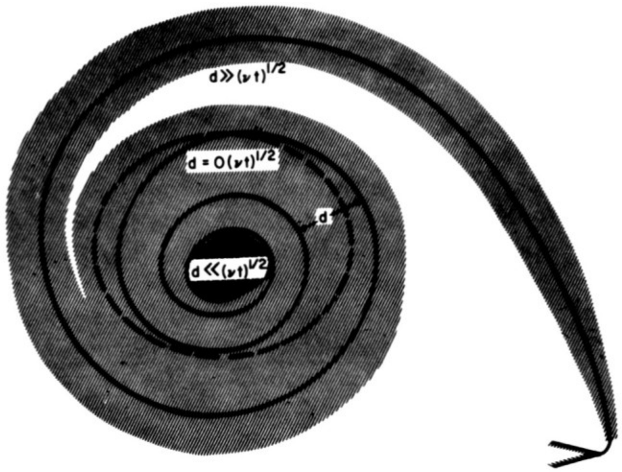

Fig. 6.

At finite Reynolds numbers, vorticity diffuses. The spiral sheet becomes a vortex layer of finite, non-zero, thickness. And its successive turns tend to merge into each other. In consequence, the small-scale structure is now determined both by convection and diffusion, the relative effects of which obviously depend on the relative magnitudes of their associated length scales d and $(\nu t)^{1/2}$. In fact, we might expect to find three distinct forms of small-scale structure characterized, respectively, by $d \gg (\nu t)^{1/2}$, $d = 0(\nu t)^{1/2}$, and $d \ll (\nu t)^{1/2}$. And since d might be expected to fall monotonically as the axis of the vortex is approached, and ultimately to vanish, all three forms of structure are likely to occur simultaneously in the same vortex if the Reynolds number Ux/ν is sufficiently large.

In a vortex of this kind there is an outer region where $d \gg (\nu t)^{1/2}$ and where, in consequence, discrete turns of the spiral remain evident. Moreover, since neighboring turns are not yet merging, diffusion proceeds without constraint, and the detailed structure of the layer itself exhibits scale effect.

Inside this outer region the turns of the spiral merge into each other as d and $(\nu t)^{1/2}$ become comparable. The radial gradient of vorticity is then much reduced, and the diffusion rate consequently falls. In effect, the close spacing of successive turns of the spiral layer applies a constraint to the diffusion process, and so tends to maintain the dominance of the convective transport mechanism long after direct evidence of its associated spiral structure has been obliterated. With diffusion severely inhibited by constraint, there is likely to be little evidence of scale effect, and something in the nature of an equilibrium structure must be developed. This region is Hall's outer core.

Further in still, in the immediate neighborhood of the axis of the vortex, d becomes vanishingly small and, however high the Reynolds number, there is always a region where $d \ll (\nu t)^{1/2}$. So the entire structure of this region is influenced by the diffusive mechanism, and is consequently subject to a marked scale effect. However, its lateral scale is small, of order $(\nu t)^{1/2}$. And it is only within this small region, which Hall calls the subcore, that diffusion is so much the dominant mechanism that the length d ceases to have real physical significance as the characteristic of an essentially convective process.

It does, of course, follow from this physical picture of the vortex structure that the boundaries between the three regions lie relatively closer to the axis of the vortex the higher the Reynolds number. But scale effect is otherwise evident only in the small-scale structure itself. And provided that the origin of the shear layer is independent of Reynolds number—as it almost always is when the separation is forced to occur at a sharp edge—we would expect to find little or no scale effect on properties of the flow associated primarily with the large-scale structure—like, for example, the strength and position of the vortex itself, and the pressure field it gives rise to on the wing. Properties of this kind are likely to depend on Reynolds number only in so far as they are influenced by the effective displacement thickness of the vortex field.

The experimental evidence appears to support these conclusions very strongly. I have already referred to the observed scale effect on axial velocity. This is illustrated in Fig. 7. In addition to this Earnshaw has observed a variation in the diameter of the subcore with axial distance x which is in satisfactory agreement with Hall's theory, according to which the diameter of the subcore varies like $x^{1/2}$ (see Fig. 8). Evidence of scale effect in the extreme outer region of the vortex again rests mainly on an observation by Earnshaw. He points out that although a significant departure from conical flow occurs, in the range of his measurements, only within the subcore, there is also some evidence of a similar departure in the outer turn of the spiral vortex layer, which becomes more sharply defined at the higher Reynolds numbers. This is precisely what we would expect if diffusion caused the local thickness of the layer to increase like x^n, where n is less than unity, while the separation of successive turns of the layer increased like x. It is also worth noting that no scale effect seems to have been observed in the forces and pressures sustained by a sharp-edged slender body.

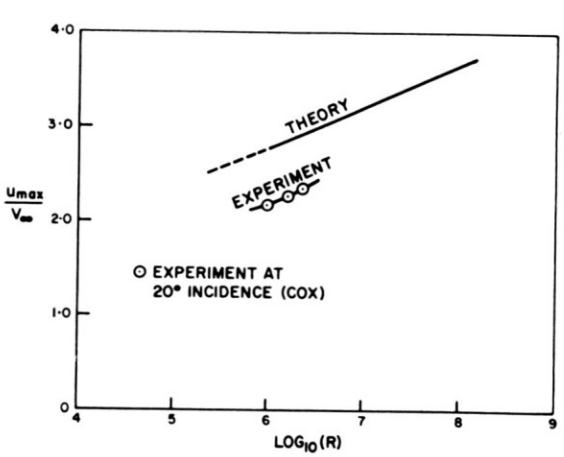

Fig. 7.

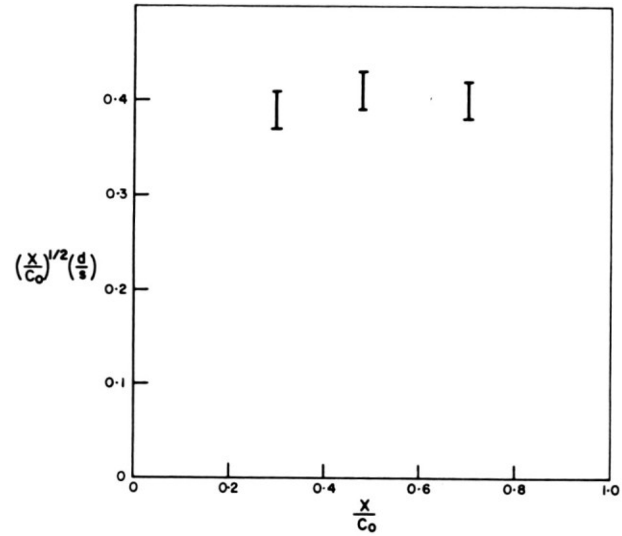

Fig. 8.

VORTEX GROWTH

The physical picture that I have tried to present sheds no direct light on the most important practical properties of the edge vortex—its strength and position, and the pressure field induced by it on a wing at given incidence in a stream of given velocity. These are functions of the rather complex interaction between the vortex and the rest of the field. And we have so far considered the vortex more or less in isolation.

However, our picture of the internal vortex structure does have some bearing on our approach to the wider problem of vortex growth. For example, it provides a basis from which to construct a suitable mathematical model of the field. In fact, in view of the apparent dominance of convection over the greater part of the vortex, it plainly lends considerable support to the spiral vortex sheet representation which has been widely assumed to hold at high Reynolds numbers. But it does so with some reservations. And it seems worthwhile to consider these in some detail, in part because they may help to explain anomalies that have been, or might be, observed in vortex behavior, and in part because they indicate possibly fruitful areas for further study.

It is important to remember that the physical argument implies that the vortex structure depends on two parameters. It depends both on the Reynolds number Ux/ν and on the rate of growth l/x of the vortex as a whole with axial distance x. The kind of structure I have inferred must be expected to hold, strictly, only when Ux/ν is very large and l/x greatly exceeds the diffusive rate of expansion of an element of vorticity relative to the rate of convection—i.e., only when $(l/x) \gg (\nu/Ux)^{1/2} \ll 1$. Only when both these conditions clearly hold can we expect to find all three of the regions characterized by the three forms of small-scale structure clearly marked in the vortex. And if they do hold I do not think there is much doubt that the spiral sheet representation provides a sufficiently close approximation so far as the vortex growth l/x and similar large-scale properties are concerned. Moreover, once this regime is reached, l/x is evidently essentially independent of Reynolds number, and is determined solely by the interaction between the vortex and the rest of the field.

But consider what happens as the Reynolds number is reduced—for example, by reducing U. We expect the boundaries between the three regions to move outwards. And eventually the extreme outer region must vanish altogether. However, when this happens, l/x may still be substantially bigger than $(\nu/Ux)^{1/2}$. As I have already pointed out, this would imply that the vortex structure is still primarily convective. And I am inclined to think that the spiral sheet model would remain essentially valid, although some scale effect might become evident experimentally as a result of the increased displacement effect of the vortex layer at lower Reynolds numbers.

As the Reynolds number is reduced further, the displacement effect must increase steadily, and lead to an increasing influence of Reynolds number on the vortex strength and position. And when the boundary of the inner core moves out far enough for it to approach the boundary of the vortex as a whole, a crucial change occurs in the mechanism controlling the vortex structure. We now have $(l/x) = 0(\nu/Ux)^{1/2}$. And so diffusion must be expected to dominate the entire structure of the vortex. The spiral sheet model can have little real meaning in this regime. For, in effect, the vortex remains submerged in the boundary layer on the wing. And there can be no justification for taking into account the one without the other.

It seems, therefore, that there are likely to be two essentially different vortex regimes:

(*a*) a viscous vortex submerged, or partially submerged, in the boundary layer, and which can be properly treated only in conjunction with the rest of the boundary-layer flow, and

(*b*) a predominantly inviscid vortex, large compared with the local boundary-layer thickness, which can be regarded as a vortex sheet (with the boundary layer neglected) subject to a relatively small modification due to viscous diffusion, much like the modification of the flow past a body at high Reynolds number due to the presence of the boundary layer.

When considering which kind of vortex is likely to occur in any given situation, it is important to remember that the relevant Reynolds number is Ux/ν, where x is a distance measured from the apex of the vortex. Thus in the case of, for example, a slender sharp-edged conical body, for which we would expect to be able to represent the leading-edge vortices by conical surfaces, there is always a region near the apex of the body where this is not justifiable, and where the vortex must be of the viscous type. There must also be an angle of incidence at which the vortices vanish altogether, whatever the Reynolds number, and a range of incidence in its neighborhood in which the vortices are again of the viscous type. Hence vortex growth, both with x at a given incidence and with incidence at a given x, necessarily begins in the viscous regime. And for a given body, it is easy to see that there is likely to be a relation between incidence α and Reynolds number Ux/ν, of the kind sketched in Fig. 9, defining the boundary between the viscous and inviscid regimes. The problem is to find what scales to put on this sketch.

In view of the apparent absence of scale effect in most experimental observations, the presence of the viscous regime near the apex of the wing is presumably not felt, to any great extent, well within the essentially inviscid region of the vortex further downstream. And I think that we are entitled to expect the vortex sheet model to be adequate for most practical purposes. But it would be dangerous, in my view, to attempt to simplify this model further, unless the simplification can be shown to leave the shape and strength of the sheet substantially unaltered in the neighborhood of the leading edge. For this part of the sheet must be expected to make a major contribution to the satisfaction of the boundary conditions applied at the leading edge. If it is ignored altogether, and all the vorticity is supposed concentrated in a line, the vortex is bound, I think, to turn out to be either too strong, or too close to the edge, or both.

SIMILARITY RELATIONS

Unfortunately, exact calculations based on the spiral sheet model seem well beyond the scope of the techniques available at the present time. The most advanced treatment of the problem is still that of Mangler and Smith,[11] and this involved quite drastic approximations to the shape of the sheet and the distribution of strength along it. Even so, the calculations were extremely complex. And there seems little immediate prospect of a much more exact treatment, especially one which is, at the same time, suitable for the study of a wide range of problems.

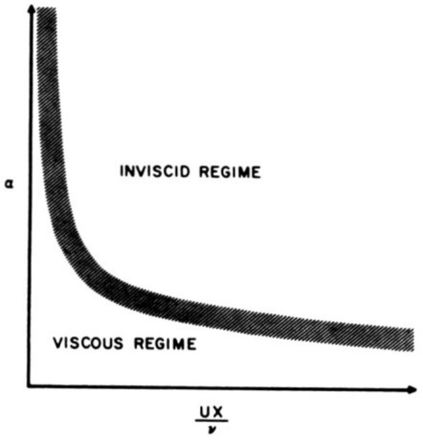

Fig. 9.

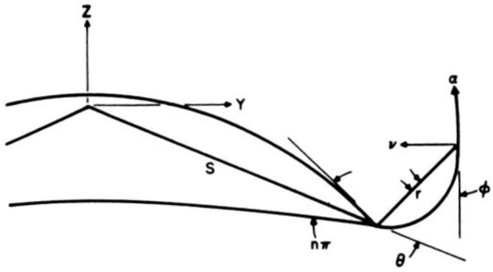

Fig. 10.

A possible alternative approach to the growth problem is to forego the attempt to calculate the field as a whole, except perhaps in certain special cases, and to seek instead similarity conditions, based rigorously on the vortex sheet model, which might enable us to generalize experimental data. This can be done, for conical bodies at least, within the framework of slender body theory, as I showed in a paper delivered at the Tenth International Congress of Applied Mechanics, in Stresa.[12]

The main idea behind this approach is that if we confine our attention to what I call the small-vortex regime—by which I mean that $(r/s) \ll 1$, where r is the radius vector from the edge to the vortex sheet, and s is the local semispan, but that the vortex is large enough to be essentially inviscid—it is convenient to regard the flow field as a vortex field superposed on the conventional attached flow which is assumed to occur in most orthodox applications of slender body theory. The boundary conditions on the body and at infinity are wholly satisfied by the attached-flow field. And so the corresponding boundary conditions for the vortex field, which includes not only the vortex sheets themselves but also their reflexions in the body, are satisfied automatically. Hence the only boundary conditions which are particular to a given problem are those that have to be applied on the vortex sheets themselves. And these are related to the known attached-flow solution.

But if the vortex is small, the only part of the attached flow that is relevant is a small region near the edge, and the effect of the other vortex can obviously be ignored. And for a sharp-edged body, the attached flow field near the edge is dominated by the singularity at the edge, and is of a form determined solely by the edge angle $n\pi$. The rest of the shape of the body determines only the orientation of this field—and hence the coordinate system, illustrated in Fig. 10, to which the vortex growth should be referred—and its magnitude. And the latter effect is embodied in two parameters, which I call S and T, which can be determined from the attached-flow solution of the problem by slender body theory.

The parameters S and T are related, respectively, to the part of the attached-flow field which scales linearly with incidence α, and the part that is independent of α. But if the factor n is either vanishingly small or close to unity, it is possible to show that only the former part need be taken into account. Then if we express the field near the edge in terms of the complex velocity, the parameter S occurs in association with α in the term

$$\frac{d\omega}{d\zeta} = - iU\alpha S\left(\frac{s}{\zeta - s}\right)^{(1-n)/(2-n)}$$

and we may regard (αS) as a kind of generalized incidence, in the sense that different bodies, all having the same edge angle $n\pi$ and the same slenderness parameter s/x, will have identical edge vortices (in relation to appropriately orientated axes) at the same value of αS.

We can then take one further step and show, by incorporating the conical similarity relation into the theory, that the similarity parameters (for $n \rightarrow 0$ and $n \rightarrow 1$) finally reduce to two: the edge-angle parameter n itself, and the generalized incidence $\alpha S/K$, where K is the slenderness parameter s/x.

Figure 11 shows how S varies with n for rhombic and biconvex circular arc cross sections, for both of which it can be written, quite simply, in terms of tabulated functions. It has also been determined for other relatively simple cross sections, including the effects of conical camber and dihedral. And, in principle, it can be determined for any cross section, once its conformal transformation into a circle is known.

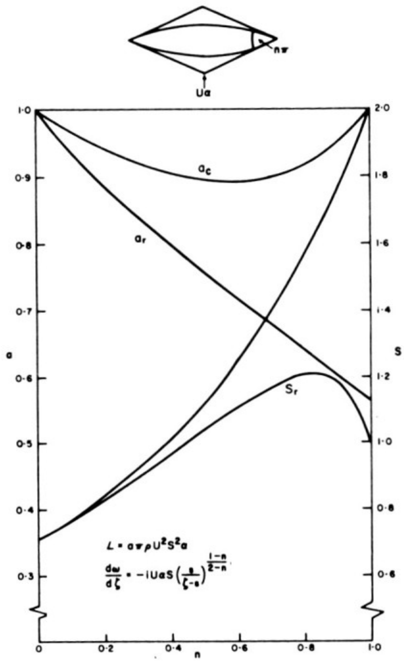

Fig. 11.

CONCLUDING REMARKS

I have tried to give a connected account of some of the more recent studies of edge vortex flows, hoping thereby to reveal both the strengths and weaknesses of our physical understanding of these flows at the present time. In particular, I have tried to show that the outline of a physical picture of the internal structure of a vortex, consistent with observation, can now be drawn. And with improved experimental techniques, to which I have briefly referred, I have no doubt that many of the details will be drawn in fairly rapidly.

I have also suggested that the spiral-sheet model of a vortex is now much more firmly based, as a result of greater understanding of its internal mechanics. But although this may give us greater confidence in that model, as a basis for the study of vortex growth, no great progress has been made in the prediction of vortex strength and position. There is, in addition, some doubt as to whether it is sufficient to represent only the leading edge vortices themselves. Secondary vortex separations are usually observed inboard of the leading edges, and there may well be a whole train of vortices of diminishing size between any primary and secondary separation.

Lastly, I have referred to the similarity relations that can be derived for small vortices in the flow past conical sharp-edged bodies, without recourse to drastic approximations to the vortex system. This, together with associated experiments, might be expected to lead to further understanding of the growth process. But its limitations have not yet been fully explored. And there is no immediate prospect of extension to nonconical fields.

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Discussor: R. Legendre, ONERA

(Après l'intervention de Mr. Maurice Roy): Observe que si le schéma établi pour la fluide parfait doit être complète de l'influence de la viscosité, il faut tenir compte de nombreux effets. La transition peut apparaître dans l'une des deux couches limites sur l'aile et se prolonger sur la nappe tourbillonnaire. Enfin, il peut y avoir séparation sur l'axe asymptatique de la nappe (vortex breakdown).

(*Author did not reply.*)